Polynomial stabilizability by optimal control.

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Plan

I **Polynomial stability and stabilization**

II **Polynomial stabilization by optimal control**

III **Inverse optimal control**

This is a joint work with C. Jammazi.

I Polynomial stability and stabilization

❶ Problems in control theory

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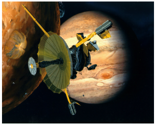
- Stabilization : Stabilize the system for make it insensitive to certain perturbations.
- Observation : Reconstruct the full state of the system from partial data.

❷ Control theory and applications

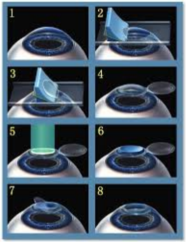
Drone

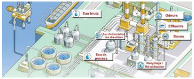
Navire



Fusée

Satellite





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$$||x(t)|| \simeq \frac{c}{t^\alpha}, \alpha > 0.$$

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$$||x(t)|| \simeq \frac{c}{t^\alpha}, \alpha > 0.$$

- Example : the scalar system $\dot{x} = -x^{2p+1}$, $p \in \mathbb{N}^*$ is polynomially stable. By simple integration we have $||x(t)|| \leq \dfrac{c}{(a\,t+b)^{\frac{1}{2p}}}$.

4 Notion of polynomial stability

Let the dynamical systems

(1) $$\dot{x}_1 = X_1(x_1, x_2),\ \dot{x}_2 = X_2(x_1, x_2),$$

$X = (X_1, X_2)$ is a continuous vector field defined on $\mathbb{R}^p \times \mathbb{R}^{n-p}$
$x := (x_1, x_2) \in \mathbb{R}^p \times \mathbb{R}^{n-p}$ is the state.

(2) $$X_1(0, x_2) = 0 \text{ and } X_2(0, x_2) = 0\ \forall x_2 \in \mathbb{R}^{n-p},$$

$x(0) = (x_1(0), x_2(0))$ is the initial condition.

5 Polynomial stability in partial sense

Definition : Appl. Math. Comput.2013

The system (1) is said to be p-rational partially stable if the following properties are satisfied

- The origin $(0,0)$ of the system (1) is Lyapunov stable :
  $\forall \varepsilon > 0,\ \exists \eta > 0 :\ (|x(0)| < \eta) \Rightarrow (|x(t)| < \varepsilon\ \ \forall t \geq 0).$
- There exist positive numbers M, k, η, r with $\eta \leq 1$ such that if $|x(0)| \leq r$ then

$$\begin{cases} |x_1(t)| \leq \dfrac{M\,|x(0)|^{\eta}}{(1+|x(0)|^{k}\,t)^{k}},\ \forall\, t \geq 0, \\ \\ \lim\limits_{t\to+\infty} x_2(t) = a(x(0)), \end{cases}$$

where $a(x(0))$ is a constant vector depending on initial conditions.

Definition

Let be the control system

$$(3) \qquad \begin{cases} \dot{x} = X(x, u) \\ X(0, 0) = 0, \end{cases}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ the control, and $X \in C^0(\mathbb{R}^{n+m}, \mathbb{R}^n)$. System (3) is p-partially rationally stabilizable, if there exists a continuous feedback $x \mapsto u(x)$ such that, for every $x_2 \in \mathbb{R}^{n-p}$, $u(0, x_2) = 0$, and $(0,0) \in \mathbb{R}^p \times \mathbb{R}^{n-p}$ is p-rationally stable for the closed loop system $\dot{x} = X(x, u(x))$.

Remark

Obviously, the case $n = p$ corresponds to complete rational stability of system (1).

- A. Bacciotti. Stability analysis based on direct Lyapunov method, chapter Lecteurs given at the Summer School on Mathematical Control Theory, pages 315-363. 2001.
- A. Bacciotti. Stability by damping control. Differential Equations and Dynamical Systems, 10 :331-341, 2002.
- A. Bacciotti and L. Rosier. Liapunov Functions and Stability in Control Theory. Communications and Control Engineering, Springer-Verlag, 2005.
- C. Jammazi and M. Zaghdoudi. On the rational stabiliy of autonomous dynamical systems. Applications to chained systems. Appl. Math. Comput. 219 (2013), 10158-10171..

Proposition : Nonlinear Analysis-2019

Consider the dynamical system (1), we assume that there exist a locally Lipschitz function $V : \mathbb{R}^n \to \mathbb{R}$, some positive constants c_1, c_2, r_1 and r_2 such that

(a) there exists $\varepsilon > 0$, such that for every x ; $|x| < \varepsilon$, V satisfies

$$c_1 |x|^{r_1} \leqslant V(x) \leqslant c_2 |x|^{r_2},$$

(b) there exist $c > 0$ and $\alpha > 0$ such that

$$D^+ V(x(t)) + cV^{1+\alpha}(x(t)) \leqslant 0. \tag{4}$$

Then $0 \in \mathbb{R}^n$ is locally rationally stable. Moreover, if the first condition holds for all $x \in \mathbb{R}^n$, then $0 \in \mathbb{R}^n$ is globally rationally stable.

Example :

(5) $$\dot{x} = -x^r,\ \dot{y} = -yx^2,$$

where $r \in \mathbb{Q}^+_{odd} \bigcap (1, +\infty)$.

$$\mathbb{Q}^+_{odd} = \{r \in \mathbb{Q}_+ : r = \frac{p}{q}, \text{ where } p \text{ and } q \text{ are odd non negative integers}\}.$$

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$$V = \frac{1}{2}(x^2 + y^2),\ \ \dot{V} = -x^{r+1} - x^2 y^2 \leqslant 0.$$

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$$W(x) = V(x, 0)$$

$\dot{W} \leqslant -cW^{(r+1)/2}$, in particular

$$|x(t)| \leqslant \frac{k}{t^{1/(r-1)}}.$$

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II Polynomial stabilization by optimal control

1. Sufficient conditions for polynomial stabilization by optimal control
2. Application to systems with drift in Vorotnikov sense
3. Example

1/1 Sufficient conditions for polynomial stability

$$\dot{x}_1 = X_1(x, u)\dot{x}_2 = X_2(x, u) \tag{6}$$

$$X_1(0, x_2, 0) = 0 \text{ and } X_2(0, x_2, 0) = 0 \; \forall x_2 \in \mathbb{R}^{n-p}. \tag{7}$$

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$\Gamma(x(0)) := \{u : u$ is admissible, $x(t) = (x_1(t), x_2(t))$ solution of (6) such $that\ |x_1(t)| \leq \dfrac{c}{t^\alpha}\ and\ x_2(t) \to c \in \mathbb{R}^{n-p}\}$.

Optimal control

Bernstein :93 Optimal feedback of nonlinear regulation problems involving a non quadratic cost functionals, the Hamilton—Jacobi—Bellman approach is used.

Haddad et al :2014 Singular control for Linear Semistabilization

L'Affalito et al :2015 Asymptotic stabilization (resp. finite-time stabilization) with respect to part of the system....

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Problem

$$(\mathscr{P}): \quad J(x(0), u^*) = \min_{u \in \Gamma} \int_0^{+\infty} L(x(t), u(t))\,dt,$$

where $L : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ is continuous and positive function.

1/3 Sufficient conditions for polynomial stabilization by optimal control

Proposition

Assume that there exist a $\mathscr{C}^1$ function $V : \mathscr{O} \times \mathbb{R}^{n-p} \to \mathbb{R}$, some positive constants $c_1,\ c_2,\ r_1,\ r_2,\ c,\ \alpha$, a continuous function $u^* : \mathscr{O} \times \mathbb{R}^{n-p} \to \mathscr{U}$ such that $u^*(0, x_2) = 0,\ \ \forall x_2 \in \mathbb{R}^{n-p}$ and

(a)

(9) $$c_1 |x_1|^{r_1} \leq V(x) \leq c_2 |x_1|^{r_2},\ \forall x_1 \in \mathscr{O},$$

(10) $$\dot{V}(x) \leq -c\, V^{\alpha+1},$$

(11) $$(\exists \eta > 0 : |x(0)| < \eta) \Rightarrow \int_0^{+\infty} |X_2(x(s), u^*(x(s)))| ds < \infty,$$

Sufficient conditions for polynomial stabilization by optimal control

(b) for every $x = (x_1, x_2) \in \mathcal{O} \times \mathbb{R}^{n-p}$: V and L satisfy

(12) $$L(x, u^*) + \nabla V(x) \cdot X(x, u^*) = 0$$

(13) $$L(x, u) + \nabla V(x) \cdot X(x, u) \geq 0.$$

Then, the system (6) is p-partially locally rationally stabilizable, and

(14) $$J(x(0), u^*) = V(x(0)).$$

Furthermore the feedback control u^* minimizes J in the sense that

(15) $$J(x(0), u^*) = \min_{u \in \Gamma(x(0))} J(x(0), u).$$

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$$J(x(0), u^*) = \min_{u \in \Gamma(x(0))} J(x(0), u). \tag{15}$$

$$u^* := argmin_{u \in \Gamma(x(0))} [L(x, u) + \nabla V(x) \cdot X(x, u)].$$

2/1 Application to systems with drift in Vorotnikov sense

$$
(16) \qquad \begin{cases} \dot{x}_1 & = & f_1(x) + G_1(x)\, u \\ \dot{x}_2 & = & f_2(x) + G_2(x)\, u, \end{cases}
$$

where $x = (x_1, x_2) \in \mathbb{R}^p \times \mathbb{R}^{n-p}$, $u \in \mathbb{R}^m$, $f = (f_1, f_2)$ defined on $\mathbb{R}^p \times \mathbb{R}^{n-p}$ and $G = (G_1, G_2) : \mathbb{R}^p \times \mathbb{R}^{n-p} \to \mathbb{R}^{p\times(n-p)\times m}$.

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(17)
$$J(x(0), u) = \int_0^{+\infty} L(x, u(t))\, dt.$$

(18)
$$L(x, u) = L_1(x) + L_2(x)\,u + u^T R\,u,$$

where $R : \mathbb{R}^m \to \mathbb{R}^{m\times m}$ is positive definite matrix-valued function.

$$\Lambda(x(0)) := \{u : u \text{ is admissible, } x(t) = (x_1(t), x_2(t)) \text{ solution of (16) and } |x_1(t)| \le \frac{c}{t^\alpha}\}.$$

Proposition

Consider the system with drift (16) with cost functional (17). We assume that there exist a $\mathscr{C}^1-$ function $V : \mathbb{R}^p \times \mathbb{R}^{n-p} \to \mathbb{R}$ and some positive constants $c_1,\ c_2,\ r_1,\ r_2,\ r_3,\ c,\ \alpha$ such that

(a) for every $x \in \mathbb{R}^n$,

(19) $$c_1\,|x_1|^{r_1} \le V(x) \le c_2\,|x_1|^{r_2},$$

(20) $$L_f V(x) - \frac{1}{2} L_G V(x)\,R^{-1}\,L_2^T(x) - \frac{1}{2} L_G V(x)\,R^{-1}(L_G V(x))^T \le -c\,V^{\alpha+1}$$

(b)

(21) $$L_2(0, x_2) = 0,\ \ \forall\ \ x_2 \in \mathbb{R}^{n-p}$$

(22) $L_1(x) + L_f V(x) - \frac{1}{4}\left[L_G V(x) + L_2(x)\right] \cdot R^{-1}\left[L_G V(x) + L_2(x)\right]^T = 0.$

Then the system (16) is p-partially rationally stable with Vorotnikov sense under the optimal feedback

(23) $$u^* := -\frac{1}{2} R^{-1}\left[L_2(x) + L_G V(x)\right]^T.$$

In addition, u^* minimizes the cost functional J in the sense that

(24) $$J(x(0), u^*) = \min_{u \in \Lambda(x(0))} J(x(0), u),$$

and

(25) $$J(x(0), u^*) = V(x(0)).$$

The model of spacecraft with two axis :

$$\left\{\begin{array}{lcl}\dot{\omega}_1 & = & \alpha_1\, u_1\\ \dot{\omega}_2 & = & \alpha_1\, u_2\\ \dot{\omega}_3 & = & \alpha_3\, u_1 + \alpha_4\, u_2,\end{array}\right. \tag{26}$$

where

- $x = (\omega_i)^T_{1\leqslant i\leqslant 3} \in \mathbb{R}^3$ is the state and denote the components of the angular velocity vector with respect to a given inertial reference frame expressed in a central body reference frame,
- u_1, u_2 are the spacecraft control moments,

$$J(x(0),\ u) = \int_0^{+\infty} \frac{1}{16}\, \alpha_1^2 |x_1|^{\frac{4k}{p}+2},\ p,k \in \mathbb{R}^*, \tag{27}$$

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$$V(x) = \frac{p}{4\,(k+p)}\,|x_1|^{\frac{2k}{p}+2},\ \ p,k \in \mathbb{R}_+^*.$$

(28)
$$\begin{cases} u_1^*(x) & = & -\frac{1}{4}\,\alpha_1\,|x_1|^{\frac{2k}{p}}\,\omega_1 \\ u_2^*(x) & = & -\frac{1}{4}\,\alpha_1\,|x_1|^{\frac{2k}{p}}\,\omega_2, \end{cases}$$

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(29)
$$|\omega_i(t)| \le \frac{\sqrt{c_1}}{t^{\frac{p}{2k}}},\ i = 1, 2,$$

if $p > 2k$, then the solutions $\omega_i(t),\ i = 1, 2$ are Lebesgue-integrable, then the state $\omega_3(t)$ converges.

III Inverse optimal control

- **Problem :**

Given a control system,

$$\dot{x} = f(x, u),$$

a specific feedback law control u^* stabilizes this system, with respect to a positive definite radially unbounded Lyapunov function V.

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- **Goal :**

to find a Lagrangian function L for which this control u^* is optimal in integral cost sense.

1. **Freeman et Kokotovic :96** Optimalité inverse pour la stabilisation robuste.

2. **Tsiotras :99** Optimalité inverse pour la stabilisation d'un satellite.

3. **Edouard et al :2014** Problème de contrôle optimal inverse avec une optimisation polynomiale.

4. **Haddad :2014, L'Afflitto al :2015, 2016** Stabilisation asymptotique (respectivement en temps fini) par rapport à une partie du système par un contrôle optimal inverse...

Proposition

Consider the system with drift (16) subject to cost functional (17). We assume that there exist a $\mathscr{C}^1-$ function V and some positive constants c_1, c_2, r_1, r_2, r_3 c, α such that the set of conditions hold

(a)

$$c_1\,|x|^{r_1} \leq V(x) \leq c_2\,|x|^{r_2},$$

for every $x \in \mathbb{R}^n$

$$L_f V(x) - \frac{1}{2} L_G V(x)\, R^{-1}\, L_2^T(x) - \frac{1}{2} L_G V(x) R^{-1} (L_G V(x))^T \leq -c\, V^{\alpha+1}(x)$$

(b)

$$L_2(0, x_2) = 0, \; \forall x_2 \in \mathbb{R}^{n-p}$$

then under the feedback

$$u^* := -\frac{1}{2} R^{-1}.\left[L_2(x) + L_G V(x)\right]^T,$$

the closed loop system (16) is partially rationally stable in Vorotnikov sense. Furthermore u^* is optimal with respect the cost functional (17), where

$$L_1(x) = u^{*\,T}(x)\,R.\,u^*(x) - L_f V(x)$$

i.e. J is minimized in the sense (24) and (25).

The model of spacecraft with one axe of symmetry :

$$\begin{cases} \dot{\omega}_1 & = & I_{23}\,\omega_2\,\omega_3 + u_1 \\ \dot{\omega}_2 & = & -I_{23}\,\omega_3\,\omega_1 + u_2 \\ \dot{\omega}_3 & = & \alpha_3\,u_1 + \alpha_4\,u_2, \end{cases} \tag{30}$$

- $x := (\omega_i)^T_{1\leqslant i\leqslant 3} \in \mathbb{R}^3$ is the state and denote the components of the angular velocity vector with respect to a given inertial reference.
- u_1, u_2 are the spacecraft control moments.
- $\alpha_3, \alpha_4 \in \mathbb{R}$.
- $I_{23} = \dfrac{I_2 - I_3}{I_1}$, I_1, I_2 and I_3 are the principal moments of inertia of the spacecraft such that $0 < I_1 = I_2 < I_3$.

6 Example

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$p > \max(1, 2k)$, ω_3 converge.

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$$|\omega_i(t)| \sim_{+\infty} \frac{c}{t^{\frac{p}{2k}}}, \quad i = 1, 2,$$

$p > \max(1, 2k)$, ω_3 converge.

$$L_1(x) = u^{*\,T}(x)\,R(x)\,u^*(x) - V'(x)\,f(x) = (I_{23}\,\omega_2\,\omega_3)^2 + (I_{23}\,\omega_3\,\omega_1)^2 + \frac{1}{2}|x_1|^{4\frac{k}{p}+2},$$

$$J(x(0), u) = \int_0^{+\infty} L(x, u(t))\,ds.$$

Thank you for your attention